1. Let be independent and identically distributed random variables with the same mean and variance , and let denote the sample mean. Tchebychev’s inequality says that the probability does not exceed for every constant , where denotes the variance of .
2. Prove that the statement

holds with a probability that does not exceed .

Then,

Finally,

1. Prove that the statement

holds with a probability that is not smaller than . NOTE: the previous statement gives the following confidence interval

for the (unknown) population mean , with a confidence level of at least and for every sample size .

Then,

However,

But,

Therefore,

1. Prove that the statement

holds with a probability that is not smaller than

Then,

And,

Therefore,

1. Let be a sample from a population , and let be a filter (that is, a random variable in Statistics, or a measurable function in Mathematics) that produces outputs (called observations in Statistics) by the formula . Let denote the average of . Does the confidence interval

cover the (unknown) population mean or not?

The confidence interval covers the population mean with a confidence percentage of 95%.